arXiv:gr-qc/0101026v1 8 Jan 2001

Energy and Momentum in Expansive Nondecelerative Universe

Miroslav Súkeník and Jozef Šima

Faculty of Chemical Technology, Slovak Technical University, Radlinského 9, 812 37
Bratislava, Slovakia

sima@chtf.stuba.sk

Abstract. Incorporation of the Vaidya metric in the model of Expansive Nondecelerative Universe allows to precisely localize gravitational energy for weak fields and obtain the components of the Einstein energy-momentum pseudotensor for strong gravitational fields. The components are identical to those calculated by Virbhadra.

Introduction

It is becoming still more obvious that the solution of several problems of cosmology and astrophysics depends on the ability to localize gravitational energy. In this area three main streams of opinions can be identified: 1) gravitational energy is localizable but a corresponding "magic" formula for its density is to be found; 2) gravitational energy is nonlocalizable in principle; 3) gravitational energy does not exist at all since the gravitational field is a pure geometric phenomenon.

One of the approaches to the localization of gravitational energy is based on a presumption that space-time may have more than four dimensions. Kaluza-Klein theories [1] seem to be the first to have proposed a solution within this approach. The corresponding D-dimensional metric has the form

$$ds^2 = g_{\mu\nu}\left(x^\mu\right) dx^\mu dx^\nu - \gamma_{ab}\left(x^a\right) dx^a dx^b \tag{1}$$

where $g_{\mu\nu}$ is the metric of four-dimensional world while γ_{ab} is the metric associated with $\mathrm{D}-4$ compact extra dimensions.

During 80's the theory of superstrings was developed, a number of dimensions in it reaching 10. An interesting solution comes from six-dimensional space model, where the gravity is localized on a four-dimensional singular string-like defect [2]. A six-dimensional metric satisfying 4D Poincaré invariance is as follows

$$ds^2 = \sigma\left(\rho\right) g_{\mu r} dx^\mu dx^\nu - d\rho^2 - \gamma\left(\rho\right) d\theta^2 \tag{2}$$

For two extraspatial dimensions the polar coordinates ρ, θ were introduced. A different approach dealing with gravitational energy-momentum density in teleparallel gravity has

led to a true space-time and gauge tensor transforming covariantly under global Lorentz transformation [3].

Attempts to localize the gravitational energy in a classic four-dimensional space-time face the problem of scalar curvature that, using Schwarzschild metric, is of zero value outside a body. The way out may lie in application of a different metric consistent with a new model of the Universe, the model of Expansive Nondecelerative Universe (ENU) [4].

The present paper is devoted to the solution of Einstein pseudotensor and, in turn, to localization and quantification of the density of gravitational field energy in four-dimensional space-time.

Background of the ENU model

The model of Expansive Nondecelerative Universe [4] differs from more frequently used inflation models in the following features:
a) Schwarzschild metric is replaced by Vaidya metric,
b) the Universe permanently expands by the velocity of light c,
c) simultaneous creation of matter and the equivalent amount of gravitational energy (which is, however, negative and thus the total value of mass-energy is constant and equal zero) occurs. Statements b) and c) can be expressed as follows

$$a = c.t_c = \frac{2G.m_U}{c^2} \tag{3}$$

$$\Lambda = 0 \tag{4}$$

$$k = 0 \tag{5}$$

where a is the gauge factor, m_U is the Universe mass, Λ is the cosmological member, k is the curvature index, and t_c is the cosmological time. Introducing dimensionless conform time, equation (3) can be expressed as

$$c.dt = a.d\eta \tag{6}$$

from which

$$a = \frac{da}{d\eta} \tag{7}$$

Friedmann equations can be then written in the form [5]

$$\frac{d}{d\eta}\left(\frac{1}{a}.\frac{da}{d\eta}\right) = -\frac{4\pi.G}{3c^4}a^2\left(\varepsilon + 3p\right) \tag{8}$$

$$\left(\frac{1}{a}.\frac{da}{d\eta}\right)^2 = \frac{8\pi.G}{3c^4}a^2\varepsilon - k \tag{9}$$

where ε is the energy density and p is the pressure. Based on (8) and (9) it follows

$$\varepsilon = \frac{3c^4}{8\pi.G.a^2} \tag{10}$$

$$p = -\frac{\varepsilon}{3} \tag{11}$$

Equations (10) and (11) represent the matter creation and the negative value of gravitational energy, respectively. It can be evidenced that in the ENU it holds [6]

$$\sum \frac{dm}{dt} = \frac{dm_U}{dt} = \frac{m_U}{t_c} = \frac{c^3}{2G} \tag{12}$$

and

$$m_U = \frac{a.c^2}{2G} \tag{13}$$

One of the corner-stones of the ENU lies in a nonstatic nature of space-time, understood here as a time change in the Universe mass, dm_U/dt (12). Solutions of some astrophysical problems have taken this nature into account (e.g. its importance for treatment of the problem of naked singularities [7] or radiating stars [8, 9] was clearly manifested. A contribution of the ENU is in the fact that instead of a general expression of dm_U/dt it offers its value as

$$\frac{dm}{dt} = \frac{m}{t_c} \tag{14}$$

Based on (14) it is obvious that metric with varying Newton potential, in which the scalar curvature is of nonzero value also outside of body, should be applied. The first to solve this problem was Vaidya dealing with radiant stars.

The density of gravitational energy ε_g is in the first approximation expressed by Tolman equation [10]

$$\varepsilon_g = -\frac{R.c^4}{8\pi.G} \tag{15}$$

where R is the scalar curvature. Using Vaidya metric and applying relation (14) we obtain [11,12]

$$R = \frac{6G}{c^3.r^2}.\frac{dm}{dt} = \frac{3r_g}{a.r^2} \tag{16}$$

where $r_g$is the gravitational radius of a body with the mass m. Relations (15) and (16) lead to the following expression for the density of gravitational field energy

$$\varepsilon_g = -\frac{3m.c^2}{4\pi.a.r^2} \tag{17}$$

The total amount of gravitational energy emitted by a body in time unit (gravitational output P_g) is given by

$$P_g = -\frac{d}{dt}\int \frac{c^4.R}{8\pi.G} dV = -\frac{m.c^2}{t_c} \tag{18}$$

In the ENU, the wavefunction of the Universe is formulated [11] as

$$\Psi_g = e^{-i(t_{Pc}.t_c)^{-1/2}.t} \tag{19}$$

where t_{Pc} is the Planck time. For the energy of gravitational quantumit follows [12] from (19)

$$|E_g| = k.T = \left(\frac{\hbar^3.c^5}{t_c^2.G}\right)^{1/4} \tag{20}$$

where T is the Universe temperature. Relation (20) documents that the density of gravitational energy and that of radiation energy were identical during the entire radiation era, i.e. the Universe had to be in thermodynamic equilibrium up to the end of the radiation era.

Energy-Momentum in ENU

In the Vaidya metric [8,9] the line element is formulated in the form

$$ds^2 = \frac{\Psi'^2}{f_{(m)}^2}\left(1 - \frac{2\Psi}{r}\right).c^2.dt^2 - \left(1 - \frac{2\Psi}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + sin^2\theta.d\varphi^2\right) \tag{21}$$

where

$$\Psi = \frac{G.m}{c^2} \tag{22}$$

It follows from (14) and (22) in the ENU

$$\Psi' = \frac{d\Psi}{c.dt} = \frac{\Psi}{a} \tag{23}$$

$f_{(m)}$ is an arbitrary function. In order it has a nonzero value, it must hold in the ENU

$$f_{(m)} = \Psi\left[\frac{d}{dr}\left(1 - \frac{2\Psi}{r}\right)\right] = \frac{2\Psi^2}{r^2} \tag{24}$$

Gravitational influence can be realized in the ENU only when the absolute value of gravitational energy density will exceed the critical energy density. Thus, if (10) and (17) are equal, then

$$R_{ef} = (R_g.a)^{1/2} = (2\Psi.a)^{1/2} \tag{25}$$

where R_{ef} is the effective gravitational range of a body with the gravitational radius R_g. Vaidya metric is applicable in the ENU if

$$r < R_{ef} \tag{26}$$

It follows from (21) that scope of Vaidya metric applicability is determined by relation

$$\Psi' = f_{(m)} \tag{27}$$

Based on (23), (24) and (27) it holds

$$A = \frac{c^4}{4\pi.G} \tag{28}$$

Equation (28) represents the condition of changing Vaidya metric to Schwarzschild metric in which the scalar curvature becomes of zero value which prevents to localize gravitational energy. This conclusion conforms with the ENU model.

The energy-momentum complex of Einstein pseudotensor adopts the form [7]

$$\theta_i^k = \frac{1}{16\pi}\left[\frac{g_{in}}{\sqrt{-g}}\left\{-g\left(g^{kn}.g^{lm} - g^{ln}.g^{km}\right)\right\}_{,m}\right]_{,l} \tag{29}$$

Vaidya metric (21) in Cartesian Kerr-Schild coordinates was solved by Virbhadra [8] who found the following components of Einstein pseudotensor (in the Virbhadra's notation, geometrized units in which the speed of light and Newtonial gravitational constant are taken $c = G = 1$). For the sake of simplicity, we use a simplified notation where:

$$A = \frac{c^4}{4\pi.G} \tag{30}$$

$$\alpha = \frac{1}{r^3} \tag{31}$$

$$\beta = \frac{\alpha}{r} \tag{32}$$

Then the pseudotensor components are as follows:

$$\theta_0^0 = -\tfrac{A.\Psi'}{r^2} \tag{33}$$

$$\theta_1^0 = -\theta_0^1 = Ax\alpha\Psi' \tag{34}$$

$$\theta_2^0 = -\theta_0^2 = Ay\alpha\Psi' \tag{35}$$

$$\theta^0_3 = -\theta^3_0 = Az\alpha\Psi' \tag{36}$$

$$\theta^1_1 = A\beta x^2\Psi' \tag{37}$$

$$\theta^2_2 = A\beta y^2\Psi' \tag{38}$$

$$\theta^3_3 = A\beta z^2\Psi' \tag{39}$$

$$\theta^2_1 = \theta^1_2 = A\beta xy\Psi' \tag{40}$$

$$\theta^3_2 = \theta^2_3 = A\beta yz\Psi' \tag{41}$$

$$\theta^1_3 = \theta^3_1 = A\beta xz\Psi' \tag{42}$$

Our calculations stemming from the ENU provide the same components as the above found by Virbhadra. In case of weak gravitational fields, Tolman equation allows to localize the density of gravitational field energy precisely (17). It is obvious that Einstein pseudotensor is compatible with Tolman equation and the ENU model.

Virbhadra has manifested [12] general applicability of Vaidya metric. The present work is aimed to provide evidence on the applicability of Vaidya metric in solving the problems of cosmology. It seems to be obvious that further progress in the field of gravitational energy localization depends of the ability to formulate a true tensor including the changes in the Universe mass.

Acknowledgement

The authors are indebted to Professor K.S. Virbhadra for reprints of his papers and valuable suggestions.

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